Where am I? An NDT-based prior for MCL

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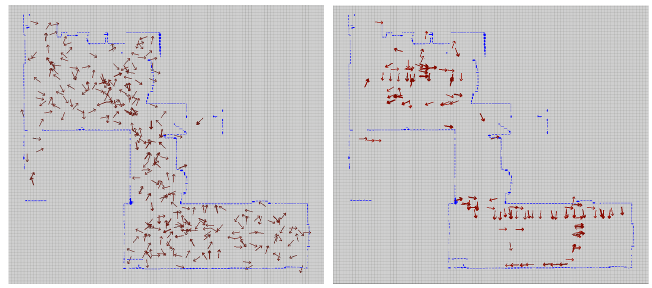
Abstract—**One of the key requirements of autonomous mobile robots is a robust and accurate localisation system. Recent advances in the development of Monte Carlo Localisation (MCL) algorithms, especially the Normal Distribution Transform Monte Carlo Localisation (NDT-MCL), provides memory-efficient reliable localisation with industry-grade precision. We propose an approach for building an informed prior for NDT-MCL (in fact for any MCL algorithm) using an initial observation of the environment and its map. Leveraging on the NDT map representation, we build a set of poses using partial observations. After that we construct a Gaussian Mixture Model (GMM) over it. Next we obtain scores for each distribution in GMM. In this way we obtain in an efficient way a prior for NDT-MCL. Our approach provides a more focused then uniform initial distribution, concentrated in states where the robot is more likely to be, by building a Gaussian mixture model over potential poses. We present evaluations and quantitative results using real-world data from an indoor environment. Our experiments show that, compared to a uniform prior, the proposed method significantly increases the number of successful initialisations of NDT-MCL and reduces the time until convergence, at a negligible initial cost for computing the prior.**

I. Introduction

Localisation is a key component of most mobile robot systems today, e.g. in field robotics, intra-logistics or assistance robots. The main focus over the past years has been to increase localisation accuracy and efficiency. Multiple solutions are already widely employed. For large outdoor environments, localisation approaches are often based on GPS, while for indoor environments, industrial localisation systems are typically based on active or passive beacons. Such systems are capable of accurate localisation, although visibility constrains and the requirement for specific infrastructure in the form of installed beacons or GPS satellites is an important drawback. These localisation methods in current practice constrain robots to operate only within limited spaces, and additionally they impose additional deployment costs.

Conversely, it is very common in the robotics community to use map-based localisation approaches without additional infrastructure. Monte Carlo Localisation (MCL) [1] is one of the most popular map-based localisation approaches and has been shown to be robust in real-world scenarios [1]–[3]. Recent works of Saarinen et al. [4] and Valencia et al. [5] have shown that by using the Normal Distributions Transform (NDT) [6], [7] for representing the environment it is possible to obtain much higher localisation accuracy with lower memory and CPU requirements, compared to occupancy grid maps [8]. Thanks to the increased accuracy (localisation error along the path is less then 3 cm [5]) NDT-MCL provides localisation good enough to fulfil industrial requirements and to be used in commercial applications. This improvement enables the development of flexible autonomous robotics systems that are independent from external infrastructure while still achieving industry-grade accuracy. However a mechanism for accurate and fast initialisation and re-initialisation of the NDT-MCL variants was so far missing. In order to achieve accurate localisation from the very start of deployment, it is necessary that the localisation algorithm converges very quickly to the true pose, which is not the case when using a uniform prior.



(a) Uniform initialization (b) GMM initialization

Fig. 1: Comparing the initial particle distribution using a uniform prior belief (a) vs an informed prior based on an NDT map (b).

The main contribution of this paper is a novel algorithm for constructing a prior for MCL from the robot's current sensor readings, exploiting the NDT map representation. In fact, the proposed initialisation method can be used together with any implementation of MCL. However, since it has been demonstrated [4] that NDT-MCL provides better pose estimates than occupancy grid based MCL and makes already use of an NDT map of the environment, we use NDT-MCL. We address the problem of initialisation and re-initialisation of NDT-MCL for cases where no external knowledge about the robot pose is available. We describe a method of building a Gaussian mixture model (GMM) representing the prior belief distribution of possible robot poses. The initial set of particles is then sampled from this GMM.

The remainder of the paper is organised as follows. Sec. II relates our work to the state of the art. Sec. III introduces our novel prior for NDT-MCL. Sec. IV describes the experimental setup and compares the localisation performance obtained with the proposed informed prior to initialisation with a uniform distribution.

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II. Related work

The Monte Carlo Localisation algorithm was first introduced by Dellaert et al. [1]. The MCL algorithm is a non-parametric Bayes filter where the belief distribution is represented as a finite set of particles. In contrast to parametric representations (e.g. Kalman filters) this has the advantage that it can represent also multimodal distributions. MCL is commonly implemented using occupancy grid maps [9] as the map representation [10].

Over the years, there have been several attempts to improve the quality of localisation with particle filters [2], [3]. Two notable recent improvements are NDT-MCL [4] and DT-NDT-MCL [5]. These approaches use NDT maps [6], [7] rather than occupancy grid maps for representing the structure of the environment and for evaluating the sensor model. The result is a significant improvement in accuracy even with rather coarsely discretised maps, which enables much more efficient mapping and localisation, both in terms of memory and CPU requirements [4].

Using standard MCL, initialisation is typically performed either by using a normal distribution centred around an initial guess of the robot pose, or by distributing particles uniformly all over the map (possibly with the addition of excluding poses that are known to intersect with obstacles in the map).

A noteworthy modification of the initial distribution of samples is presented in the work of Yee et al. [11]. In this work a regular grid of positions over the map is constructed. To compute the distance between grid points authors uses Monte Carlo tests to compute the error statistics as a function of separation. Next, the authors compute the most likely orientation for each grid point and compute the likelihood for it. This likelihood is later on used as a weight for a Gaussian associated to respective grid point. Moreover authors assume that each one has the same isotropic covariance. In this way they obtain a Gaussian mixture model which is later used for initialisation and re-initialisation. The basic idea of using Gaussian Mixture Model shows some similarities with method introduced in following paper, however idea of Yee et al. is closer to uniform initialisation. The grid points are distributed all over the map and later on the MCL filtering step is executed to implicitly define contributing particles. In contrast, our approach cuts the search space by defining areas of interest based on initial observations which later are evaluated. Moreover, the approach of Yee et al. uses the strong assumption that all the distributions are identical and isotropic.

Oh et al. [12] present a method to incorporate additional information in particles weights. Their method splits the map into regions and associates to each one a factor that describes the probability of the robot being inside that area. This approach introduces an additional bias that decreases the likelihood of particles in less likely areas (e.g., it is more likely that a robot is on the street than inside a wall). The major drawback of this method is the fact that this additional information is stored in separate static layer and has to be rebuilt each time some property of the environment changes. Moreover, any error in this layer might cause undesired behaviour of the filter by favouring some particles based only on their location even if they support the wrong hypothesis.

Dual MCL or Mixture MCL [2] suggests to invert localisation problem. Instead of first computing the new samples based on motion and then adjust its belief factor using observations authors suggest to sample form distribution based on observation and then adjust the importance factor based on the previous position of the robot. A similar but more recent approach is the observation-driven Bayes filter of He and Hirose [13]. Compared to these approaches, using NDT maps to generate poses from observations for the initial distribution is rather straight-forward. In comparison, He and Hirose [13] require pre-caching four meta-map representations and approximately one second of processing time per frame. Another interesting contribution towards implementing mixture MCL is the work of Elinas and Little [14]. However, in this work the authors employ stereo vision for localisation purposes and define a map as a set of SIFT features.

Instead of using SIFT features, we exploit the Normal Distributions Transform (NDT) environment representation. This method was introduced by Biber et al. [6] and later on extended to three dimensions by Magnusson et al. [7]. NDT is a piece-wise continuous representation, which represents space as a set of normal distributions, as opposed to occupancy grids, which represent space as a set of binary random variables. Moreover thanks to the extension by Saarinen et al. in [15] NDT Occupancy Map is able to store explicitly information about free and explored space within the environment, which is an additional asset in distributing samples over environment.

III. Prior distributions

We will now describe our informed NDT-based prior for MCL as well as the baseline uniform distribution.

A. Uniform distribution

As a baseline to evaluate the performance of our proposed approach we will use a uniform distribution of particles over the map. This approach was already discussed in the seminal work of Dellaert et al. [1].

To obtain a uniform distribution of particles over the map we use a two-step process. In the first step we uniformly draw one cell from the set of all unoccupied cells in the NDT grid (that is, all cells that do not contain a Gaussian representation of the local surface shape). In the second step we uniformly draw a position and orientation in the given cell. We repeat these two steps until we have acquired the desired number of particles. Thanks to this approach we make sure that all particles are placed in free space in the map, while keeping the execution time fixed.

B. Particle generation from GMM

In this section we will describe how to build the Gaussian mixture model (GMM) representing the prior belief distribution of the robot and how to obtain the initial set of particles from the GMM.

The procedure is as follows:

1) **Obtain promising poses -** NDT represents the global map as a set of Gaussians: $M^G_{NDT} = \{\mathcal{N}(\mu^G_j, \Sigma^G_j)\}^{N_G}_{j=1}$ To represent our current observation which is a set of n two-dimensional point samples $z = \{\mathbf{p}^z_i(x_i, y_i)\}^n_{i=1}$, we also use NDT: $\bar{Z} = \{\mathcal{N}(\mu^Z_j, \Sigma^Z_j)\}^{N_{\bar{Z}}}_{j=1}$ To build a set of possible poses we first compute the Cartesian product of the global map and the current observation: $M^G_{NDT} \times \bar{Z} = \{(g, \bar{z}) | g \in M^G_{NDT} \wedge \bar{z} \in \bar{Z}\}$ In this way combine each Gaussian from observation with each Gaussian from map. Next, for each pair $(g, \bar{z})$, we compute the pose $\mathbf{q}$ of the robot with respect to g. To solve this problem first we have to find the transformation, which will align eigen vectors with highest eigens value in $\bar{z}$ and g. Then we apply this transformation on robot pose $\mathbf{q}_{\bar{z}}$ and as a result we get $\mathbf{q}$. In this process we obtain a set $\mathcal{Q} = \{\mathbf{q}_i(x_i, y_i, \theta_i)\}^{N_z N_g}_{i=1}$ of possible poses of the robot in the coordinate frame of the global map. The size of $\mathcal{Q}$ depends on the size of the environment, the sensor field of view and the map resolution. In experiments we have observed that for an environment of size 25 x 25 m and resolution of 0.2 m number of the elements of $\mathcal{Q}$ is between 1869 and 3900, for 0.5 m is between 1755 and 2516, and for 1.0 m is between 803 and 1371.

In Fig. 3 we can see how initial set of hypotheses is generated. We can see that the current observation contains two distributions (blue and yellow). We align each Gaussian from the observation with a Gaussian in the map. Since we know what is the robot pose with respect to each Gaussian, we can transform robot pose to the global coordinate frame and obtain a set of possible poses. In Fig. 4 we can see a visualisation of all possible hypotheses obtained during one such initialisation.

2) **Build GMM -** The set of poses obtained in the previous phase implicitly mark regions of interest. To estimate the likelihood of those map regions we will generate a GMM in pose space. First we split the *state space* into a regular voxel grid $\mathcal{V} = \{v_j\}^{N_V}_{j=1}$. For simplicity of further discussion we assume that each voxel is a set of all possible poses within predefined ranges: $v_j = \{(x, y, \theta) | x \in [x^j_{min}, x^j_{max}) \wedge y \in [y^j_{min}, y^j_{max}) \wedge \theta \in [\theta^j_{min}, \theta^j_{max})\}$.For each voxel v_j that contains pose particles we estimate the corresponding normal distribution in the following way:

$$\mu = \frac{1}{n} \sum_{i=1}^{i=n} \mathbf{q}_i \tag{1}$$

$$M = [\mathbf{q}_1 - \mu ... \mathbf{q}_n - \mu] \tag{2}$$

$$\Sigma = \frac{1}{n-1} M^T M \tag{3}$$

In this way we obtain a Gaussian mixture model representing an informed prior on the poses: $M_{GMM}(\mathcal{Q}) = \{\mathcal{N}(\mu_j, \Sigma_j)\}^N_{j=1}$ where $N \leq |\mathcal{Q}|$ and $N \leq |\mathcal{V}|$. The next step is to estimate the weight w_j of each distribution in the set $M_{GMM}(\mathcal{Q})$. As a weight we will use the L_2 likelihood of the current observation at the mean pose, given the global map. The pose likelihood is computed as in Saarinen et al. [4]: $w_j = (\sum_{i=1}^{N} L^i_2)^{-1} L^j_2$ L_2 likelihood is the likelihood that the robot has a particular pose because it is consistent with several parts of the observation.

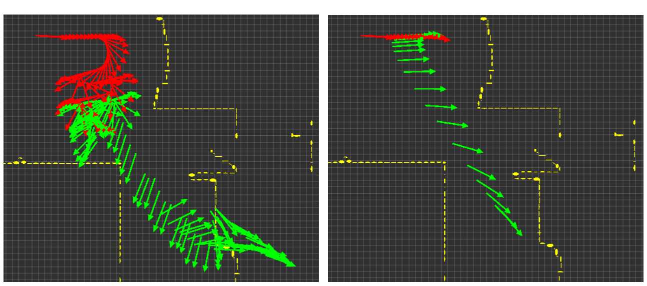
$$L^j_2(\bar{Z} | \mathbf{q}_j, M^G_{NDT}) = \sum_{i=1}^{N_G} \sum_{k=1}^{N_{\bar{Z}}} d_1 \exp(-\frac{d_2}{2} \mu^T_{ik} (R_j \Sigma^G_i R^T_j + \Sigma^Z_k)^{-1} \mu_{ik}) \tag{4}$$

where $\mu_{ik} = R_j \mu^Z_i + t_j - \mu^m_k$ and d_1 and d_2 are scaling factors.

Each pose $\mathbf{q}_j$ can be represented as a rotation matrix R_j and translation t_j with respect to the global coordinate frame. L^j_2 represents the likelihood of the current observation represented as NDT $\bar{Z}$ given the global map and the state $\mathbf{q}_j$.

Fig. 5 shows the means of all distributions of the GMM generated from the distribution shown in Fig. 4.

3) **Sampling -** The final step is to draw a set of initial particles from the GMM. The probability of drawing a sample $\mathbf{q}$ can be expressed as a sum of n weighted Gaussians: $p(\mathbf{q}) = \sum_{j=1}^{n} w_j \mathcal{N}(\mathbf{q} | \mu_j \Sigma_j)$. In this work we assume that obtained GMM is sparse therefore we can approximate this probability in the following way: $p(\mathbf{q} | \mathbf{q} \in v_j) = w_j \mathcal{N}(\mathbf{q} | \mu_j \Sigma_j)$ This approximation allows us to build a simple two step sampling algorithm. In the first step we draw a voxel according to its weight w and in the second step we draw the pose according to the normal distribution within the voxel.



(a) Uniform initialisation (b) GMM initialisation

Fig. 2: Track of convergence (500 particles, cell size = 0.2[m]) - ground truth (red), NDT-MCL localisation estimate (green). We can see here how many localisation updates are necessary to reach correct pose estimate.

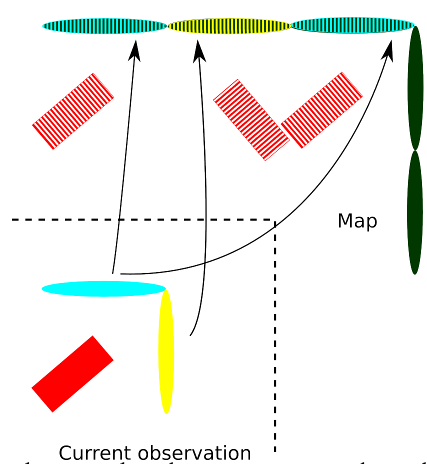
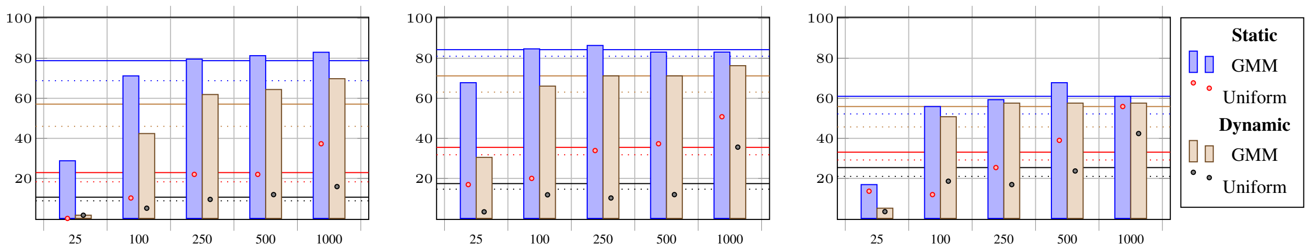
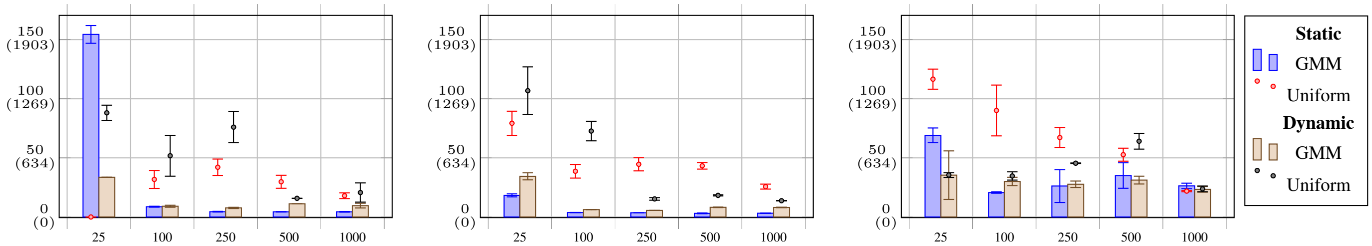


Fig. 3: Simple example where we can see how does aligning Gaussians from observation with ones from map leads to initial set of hypotheses.



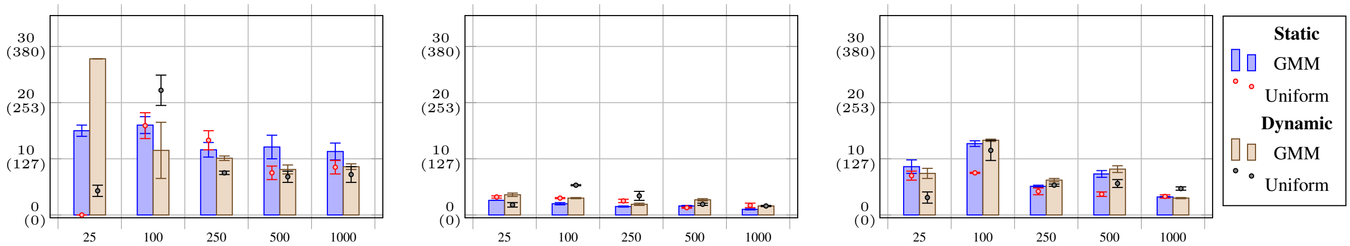
(a) map resolution 0.2 m (b) map resolution 0.5 m (c) map resolution 1.0 m

Fig. 6: Success rate [%] (dotted line - average success rate, solid line - the average success rate for 4 best cases)



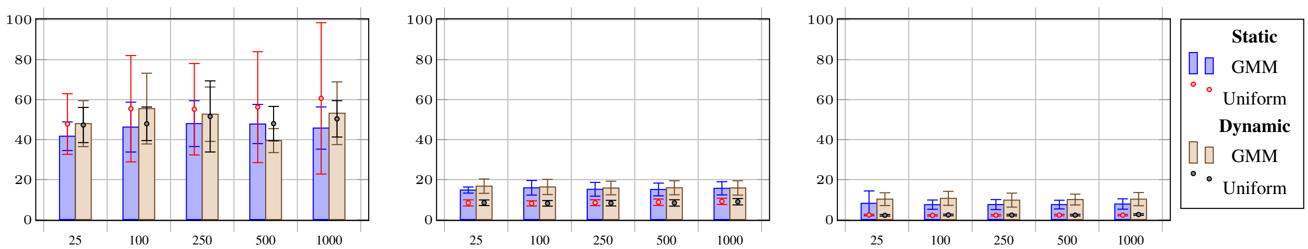
(a) map resolution 0.2 m (b) map resolution 0.5 m (c) map resolution 1.0 m

Fig. 7: Time until correct pose estimate [s (# of updates)]



(a) map resolution 0.2 m (b) map resolution 0.5 m (c) map resolution 1.0 m

Fig. 8: Time until convergence [s (# of updates)]



(a) map resolution 0.2 m (b) map resolution 0.5 m (c) map resolution 1.0 m

Fig. 9: Time for computing the prior distribution [ms]

IV. Experiments and Results

A. Experimental Setup

We have evaluated the approach presented in Section III-B using two data sets, recorded in a static and a dynamic environment. The static data set is publicly available[1]. In the test data sets, the robot traverses a closed loop (see Fig. 10) multiple times with velocity of 1 m/s in an indoor environment. Both data sets were collected in the basement of Örebro university using a commercial Automatically Guided Vehicle (AGV) system from Kollmorgen Automation AB. A Master Controller (VMC 5000) controls the vehicle along predefined trajectories. The ground truth was obtained with a commercial infrastructure-based positioning system, which tracks wall-mounted reflectors using a rotating laser. After setup and calibration, this system provides accurate (according to its specification accuracy should be approx. 1 cm or less) position information. For infra-structure free localisation we use a LIDAR with field of view of 270 degrees and range of 18 m. The data set covers a $25\,\mathrm{m} \times 25\,\mathrm{m}$ area. In both cases the robot was travelling along the same predefined path with the same velocity. To emulate a dynamic environment, we have asked a group of people to not only move around in the environment, but also on purpose to disturb the localisation

[1] Data sets are available under: http://mrolab/datasets.html.

process by changing the shape of the environment with panels or even to occlude the laser with them.

The goal of the experiment was to investigate how using an informed prior (see Fig. 1) will affect global localisation. The comparison in this paper is done between uniform initialisation of NDT-MCL and GMM initialisation of NDT-MCL, as described in Section III.

We have chosen 60 random points along the path which represent different starting positions for the localisation process. We use the following four evaluation criteria:

1) Success rate - how many times the robot manages to localise itself correctly. We consider that the robot has localised it self correctly if the error is less then 10 cm.
2) Initial localisation time - how long does it take to minimise the localisation error with respect to ground truth. In case of localisation failure the measurement was discarded.
3) Convergence time - how long it takes before the value of one standard deviation is less then 10 cm and 5 degrees. This metrics was computed only for the cases when the localisation was performed successfully.
4) Computation time - how much time it takes to generate the prior.

We have tested five particle populations sizes (25, 100, 250, 500, 1000 particles) for maps of three different resolutions (0.2 m, 0.5 m, 1.0 m). For the coarsest map (resolution 1m), the resolution of the voxel grid in pose space was 1.5 m and $\frac{\pi}{2}$ radians, and for the other two map resolutions the pose voxel grid was 0.5 m and $\frac{\pi}{2}$ radians. Please recall that we have been performing our tests using NDT maps and evaluating two different priors for NDT-MCL. If we would use a regular occupancy or octomap it would be impossible to achieve accuracy below 10 cm for maps with grid cells as big as 0.5x0.5 m^2 or 1.0x1.0 m^2 [4].

B. Results

In Fig. 6 we can see that, as long as the map resolution is sufficient enough for accurate localisation, we achieve a high success rate using the GMM prior even with a relatively small number of particles. In the best case the success rate was as high as 86% for the static environment and 76% for dynamic. The best result obtained with a uniform distribution for population containing 1000 particles is only as high 56%. It is worth noticing that for a static environment above some threshold (in our experiments around 100 samples), the average success rate is stable and does not change much when changing the number of particles and depends mainly on the map resolution. In case of the dynamic environment we can observe that the success rate increases with the number of particles, however, the success rate is high for each population equal or bigger than 100 particles. Also for dynamic environments the success rate is higher for initialisation using GMM than with uniform distribution. In Fig. 6 the average success rate across all populations is marked for a given map resolution with a dotted line. The average success rate for the four biggest sample populations is marked with a solid line. We can see that the average success rate with an NDT-based prior is always higher then the initialisation with a uniform prior. This observation is true both in static and dynamic environment.

Another interesting feature of the NDT-based prior is that it allows to localise quickly (see Fig. 7). In the best case the average localisation time was as short as 3 seconds (or 38 updates of the particle filter), while initialisation based on a uniform distribution was never faster then 18 seconds (228 updates). All the timing plots in Figs 7–9 show the results only for the cases where the filter succeeded in localising. As a consequence, the plot for 25 uniformly distributed particles in Fig. 7(a) shows zero seconds, because none of those runs succeeded. In Fig. 7 it is visible that increasing the size of the particle population does not change significantly time before successful localisation. The time needed by both distributions to converge is comparable and usually low, despite of that time needed to localise correctly is significantly shorter for proposed prior then for uniform one. The result of this time difference is visible in Fig. 2. Where the pose estimate is *following* the correct estimate.

In Fig. 9 we can see that in most cases computing the GMM-based prior only takes a few milliseconds more than computing a uniform prior. If we compare this value against the average time between two laser scans in this data set, which is 70 ms, we can assume that it is possible not only to use this method

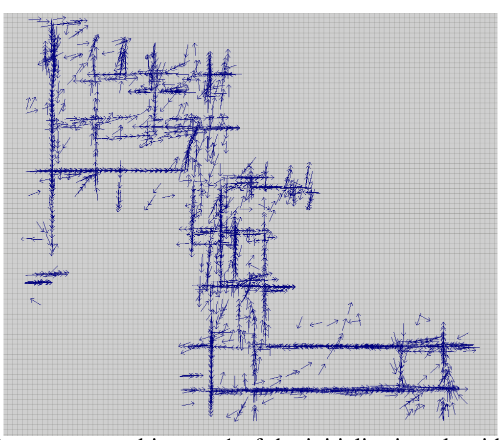


Fig. 4: Poses generated in step 1 of the initialisation algorithm, from which the GMM is created in step 2.

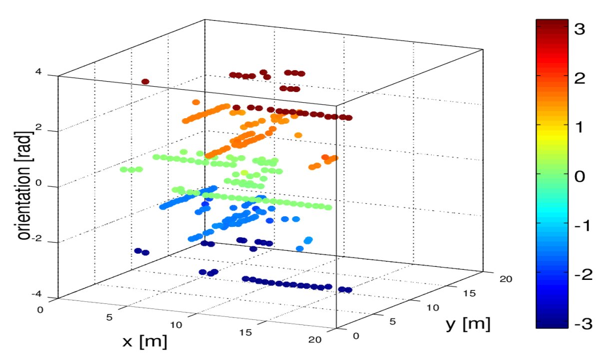


Fig. 5: An example of set of mean values for each component in GMM in map with resolution 0.2 m.

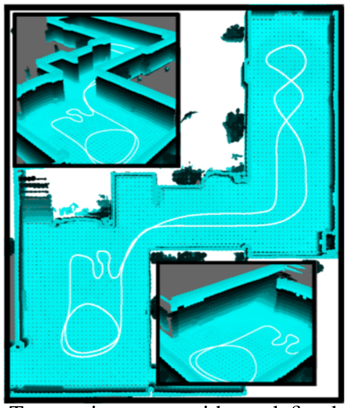


Fig. 10: Test environment with predefined path [4].

for initialisation but also for re-initialisation. If we want to use NDT-based prior for non-NDT MCL we have to remember that additional time will be required to build map of environment using NDT.

To evaluate the usefulness of the GMM method in case of re-initialisation, we have manually triggered resets of the localisation system during the robot runs in static environment. We have observed that after a high spike in the localisation error (at the moment of reset) the error drops again. During the test run we have reset the system 26 times and managed to recover in 20 cases when using the informed prior. For the system using uniform distribution for re-initialisation the success rate was only 7 recoveries for 26 resets.

V. Conclusions

In this paper we have introduced a method for constructing an informed prior for MCL. Based on a map of the environment we build a GMM which represents likely poses of the robot. Then we sample from the GMM distribution to obtain the initial set of particles.

The method introduced in this paper shows a way to implicitly define regions of interest, by removing areas that have no support from current observations. This is a major improvement in comparison to methods which are evaluating poses all over the given map. Moreover, the method builds and maintains a probabilistic model of the robot pose estimate on the fly. Therefore it does not require any additional process after obtaining the map of environment, such as assigning environment classes to regions of the map [12]. It also makes the method flexible enough to incorporate new information acquired by the sensor. Although such observation-driven priors have been used for other MCL implementations previously, this is the first implementation of a method for generating an informed prior for MCL. The main motivation for our method is the recent demonstrations [4], [5] of using NDT-MCL to achieve superior accuracy in dynamic industrial environments while maintaining a small memory footprint and low CPU requirements.

We also have performed a series of experiments both in a static and a dynamic environment demonstrating that the proposed method is able to perform global localisation with fewer particles, in comparison to the baseline uniform distribution. Moreover we have shown that an NDT-MCL particle filter initialised with NDT-based prior converges faster than when using a uniform distribution. We have also demonstrated that the significantly decreased localisation time (as measured in number of seconds or laser scans after initialisation) can be achieved with only a negligible one-time computational cost of a few milliseconds for generating the prior.

VI. Future work

In future work, we will extend the GMM method to 3D and also evaluate its performance with sensors that have a smaller field of view (e.g., RGB-D cameras).

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